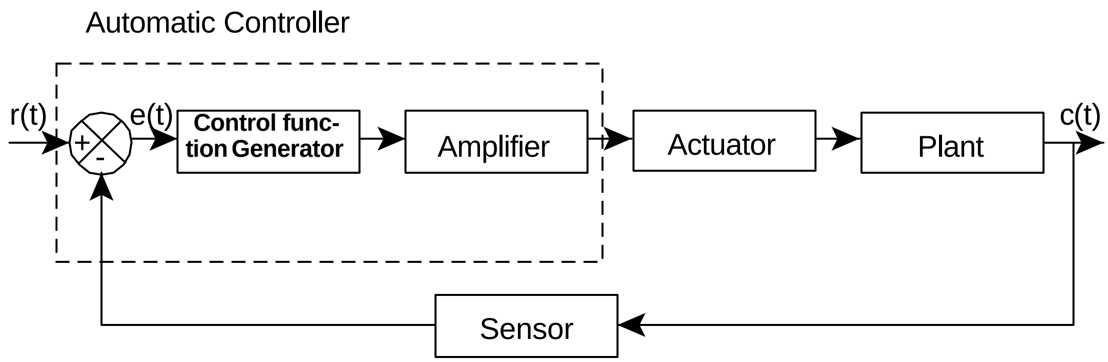
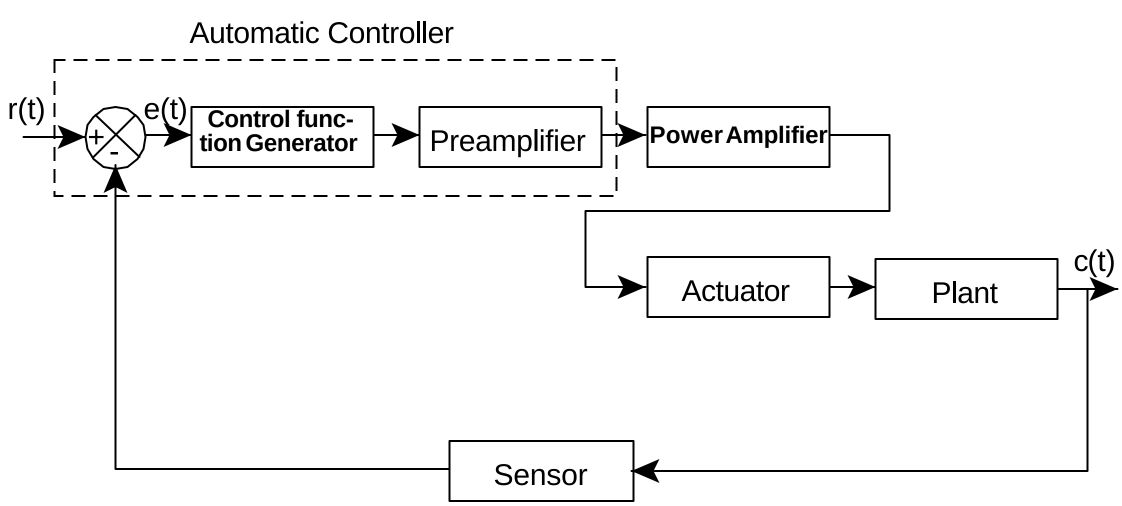
GENERAL STRUCTURE OF CONTROLLERS

An automatic controller generally consists of a comparator, a control function generator, and an amplifier.



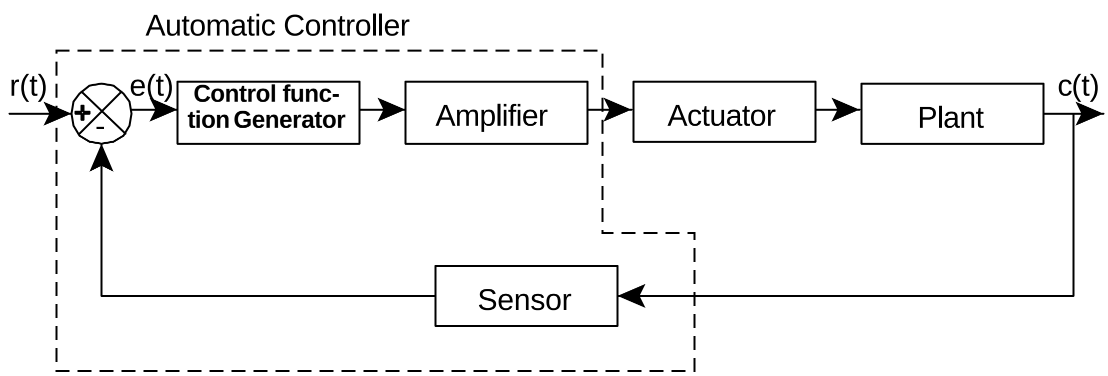
GENERAL STRUCTURE OF CONTROLLERS

In cases where the actuator requires a great deal of power, a preamplifier would replace the amplifier as part of the automatic controller, and a power amplifier would be provided to power the actuator as shown below.



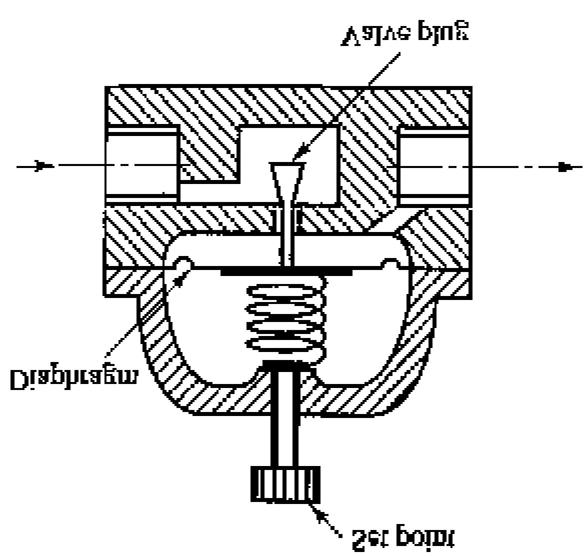
GENERAL STRUCTURE OF CONTROLLERS

In some cases of simple controllers, the comparator, control function generator, amplifier and sensor may be integrated together.



GENERAL STRUCTURE OF CONTROLLERS

An example of such an integrated controller is the pressure regulator shown below. The turn knob is used to provide the set point. The spring and diaphragm combination act as an integrated comparator, sensor and amplifier.



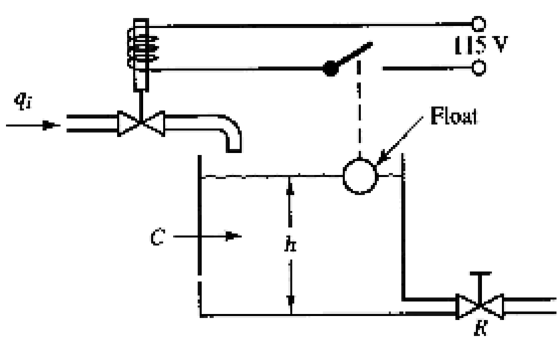
BASIC CONTROL ACTIONS

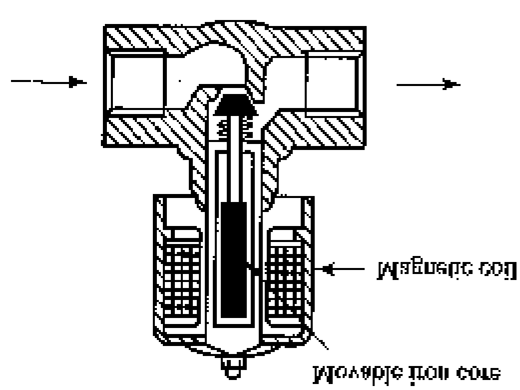
The follwing type of controll actions represent the basic set, and the ones most commonly used in industry. These are by no means the only control actions available.

- Two position controllers (also known as on-off controllers)
- Proportional controllers
- Integral controllers
- Proportional-plus-integral controllers (PI controllers)
- Proportional-plus derivative controllers (PD controllers)
- Proportional-plus-integral-plus-derivative controllers (PID controllers)

BASIC CONTROL ACTIONS

Two-Position Controllers



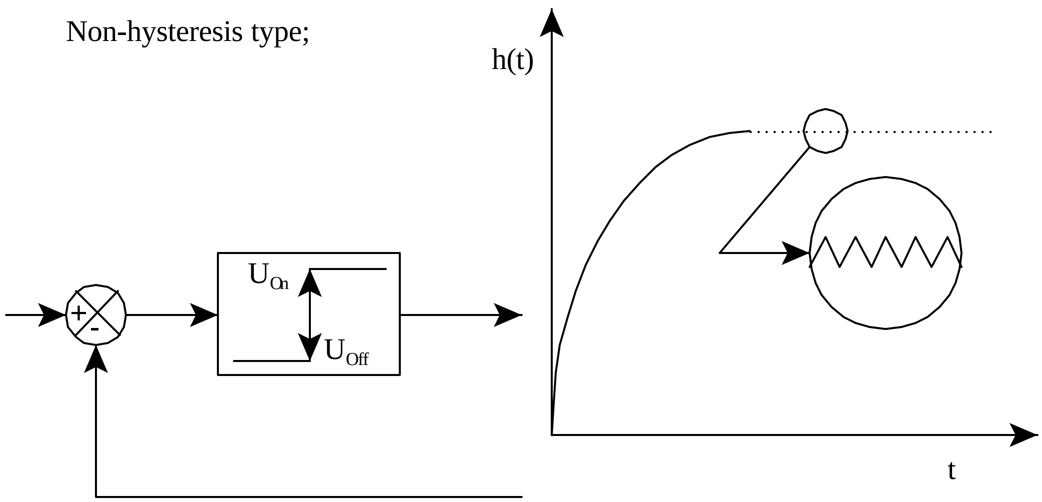


BASIC CONTROL ACTIONS

Two-Position Controllers

Design of two-position controllers:

Non-hysteresis type;

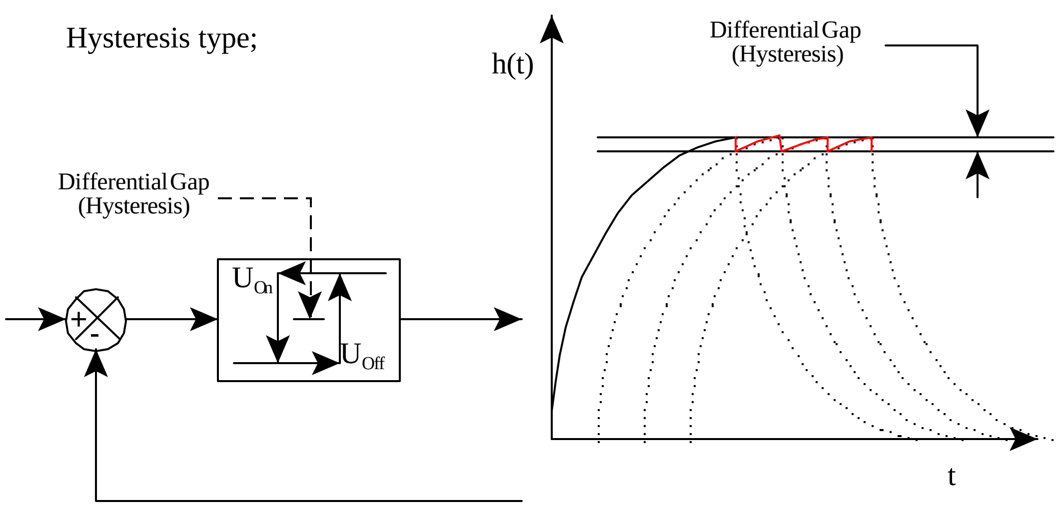


BASIC CONTROL ACTIONS

Two-Position Controllers

Design of two-position controllers:

Hysteresis type;



BASIC CONTROL ACTIONS

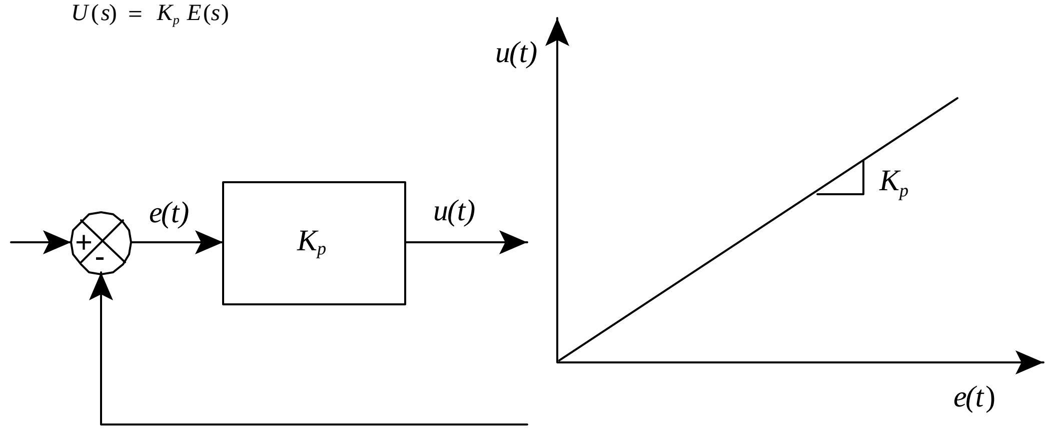
Proportional Controllers

These are by far the most commonly used industrial controllers.
The relationship between the error signal and the controller output is given by:

$$u(t) = K_p\, e(t)$$

or

$$U(s) = K_p\, E(s)$$



BASIC CONTROL ACTIONS

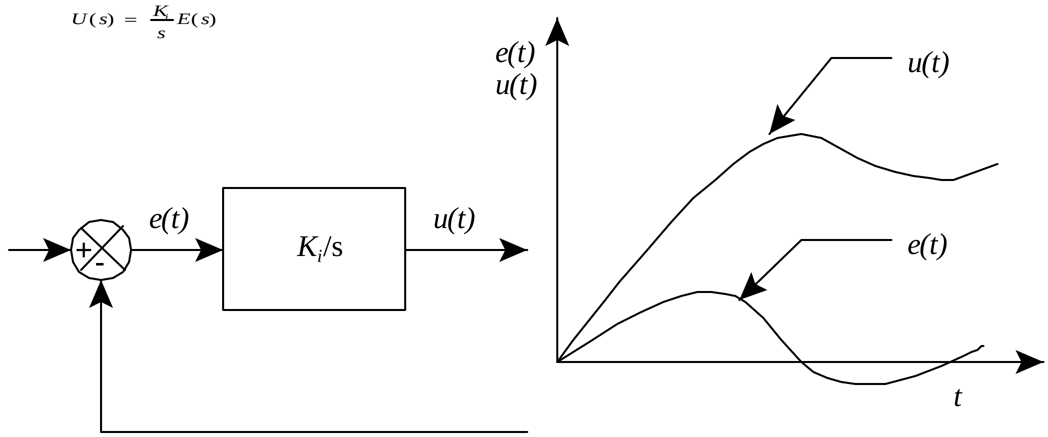
Integral Controllers

Integral controllers are used alone for the control of systems with transfer functions that have no integrator component, 1/s.
The relationship between the error signal and the controller output is given by:

$$u(t) = K_i \int_0^t e(t)\, dt$$

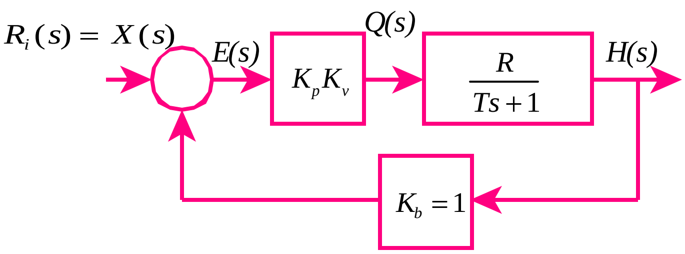
or

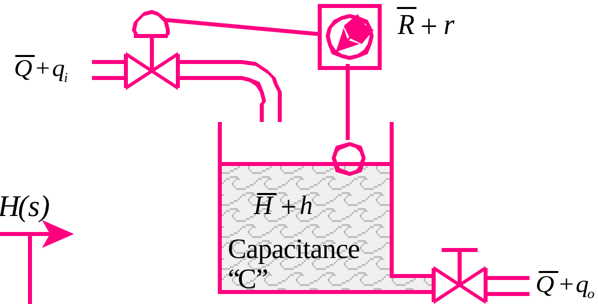
$$U(s) = \frac{K_i}{s} E(s)$$



First Order System Control
(Liquid Level Controller)

Shown below is a first order system controller and its block diagram:





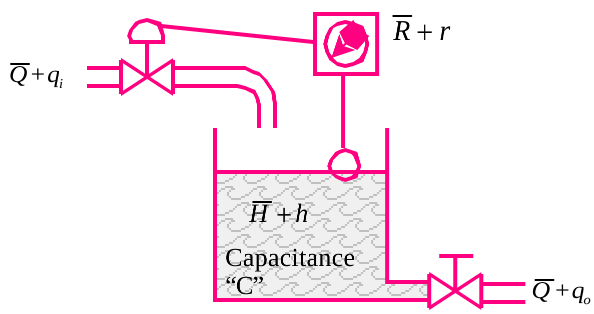
The transfer function for this system is given by:

$$\frac{H(s)}{X(s)} = \frac{K_p K_v R}{Ts+1+K_p K_v R K_b} = \frac{K}{Ts+1+K}$$

First Order System Control
(Liquid Level Controller)

For a step input 1/s the response of the system is given by:

$$H(s) = \frac{K}{Ts+1+K}\frac{1}{s}$$



Expanding *H(s)* using partial fractions give:

$$H(s) = \frac{K}{1+K}\frac{1}{s} - \frac{K}{1+K}\frac{1}{s+(1+K)/T}$$

Taking the inverse Laplace transform of both sides we get;

$$h(t) = \frac{K}{1+k}(1-e^{-t/T_1}) \qquad \text{where } T_1 = \frac{T}{1+K} \qquad \text{for } t \ge 0$$

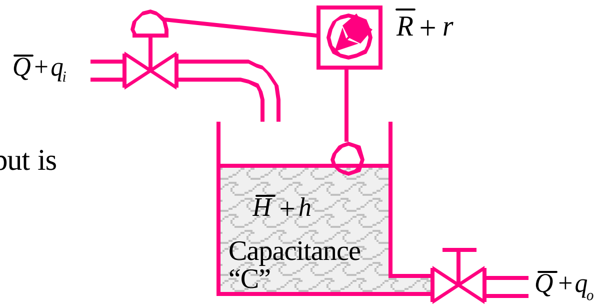
As $t \to \infty$ $\quad h(\infty) = \frac{K}{1+K}$ and a steady state error "offset error" of magnitude *K/(1+K)* persist

BASIC CONTROL ACTIONS

Integral Controllers

The transfer function of the liquid level system shown in the schematic diagram below was derived earliar for the case when the controller is a proportional one and is given by:

$$\frac{H(s)}{X(s)} = \frac{K_p K_v R}{Ts + 1 + K_p K_v R K_b} = \left.\frac{K}{Ts+1+K}\right|_{K_b=1}$$



The error of the system due to a unit step input is given by:

$$E(s) = \frac{Ts+1}{Ts+1+K}\,\frac{1}{s}$$

The steady state error is hence given by:

$$e_{ss} = \lim_{s \to 0}\left[s\,\frac{Ts+1}{Ts+1+K}\,\frac{1}{s}\right] = \frac{1}{1+K}$$

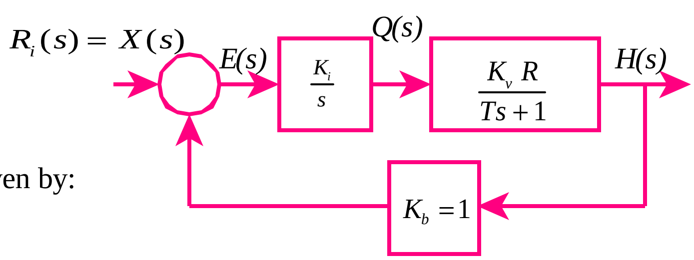
BASIC CONTROL ACTIONS

Integral Controllers

For the case where the controller is an integral one, the block diagram of the system is shown below together with its transfer function.

$$\frac{H(s)}{X(s)} = \frac{K_i K_v R}{Ts^2 + s + K_i K_v R K_b} = \left.\frac{K_i K}{Ts^2+s+K_i K}\right|_{K_b=1}$$

The error of this system due to a unit step input is given by:



$$E(s) = \frac{Ts^2+s}{Ts^2+s+K_i K}\,\frac{1}{s}$$

The steady state error is hence given by:

$$e_{ss} = \lim_{s \to 0}\left[s\,\frac{Ts^2+s}{Ts^2+s+K_i K}\,\frac{1}{s}\right] = 0$$

BASIC CONTROL ACTIONS

Proportional-plus-Integral Controllers

A proportional-plus-integral controller can also reduce the steady state error of the liquid level system to zero. The form of that controller is shown below;

$$u(t) = K_p e(t) + \frac{K_p}{T_i}\int_0^t e(t)\,dt$$

or

$$U(s) = K_p(1 + \frac{1}{T_i s})\,E(s)$$

Replacing the integral controller used before with the PI controller results in the following transfer function:

$$\frac{H(s)}{X(s)} = \frac{As + B}{T_i T s^2 + (T_i + A)\,s + B}$$

where;

$$T_i K_p K_v R = A$$
$$K_p K_v R = B$$

and $K_b = 1$ as before

BASIC CONTROL ACTIONS

Proportional-plus-Integral Controllers

The error of the system due to a unit step input is given by:

$$E(s) = \frac{TT_i s^2 + T_i s}{TT_i s^2 + (T_i + A)s + B}\;\frac{1}{s}$$

The steady state error is hence given by:

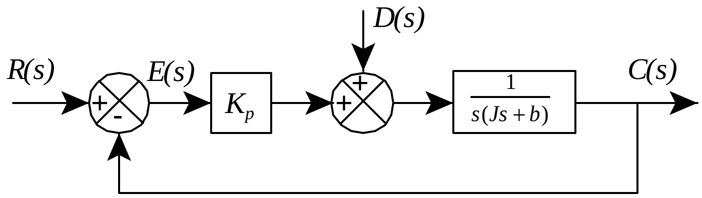
$$e_{ss} = \lim_{s \to 0}\left[s\,\frac{TT_i s^2 + T_i s}{TT_i s^2 + (T_i + A)s + B}\;\frac{1}{s}\right]$$
$$= 0$$

BASIC CONTROL ACTIONS

Proportional-plus-Integral Controllers

Consider the <u>proportionally controlled</u> second order system shown below. The system is subjected to a disturbance *D(s)* as can be seen.

Assuming *R(s)* = 0, the transfer function between *C(s)* and *D(s)* is given by:



$$\frac{C(s)}{D(s)} = \frac{1}{Js^2 + bs + K_p}$$

In the absence of *R(s)* the error of the system *E(s)* is equal to - *C(s)*. Assuming a step disturbance with magnitude A_d, the steady state error of the system is given by:

$$e_{ss} = \lim_{s \to 0}\left[s \frac{-1}{Js^2 + bs + K_p} \frac{A_d}{s} \right] = -\frac{A_d}{K_p}$$

BASIC CONTROL ACTIONS

Proportional-plus-Integral Controllers

If we replace the proportional controller with a PI controller, the transfer function between the output and the disturbance would be given by:

$$\frac{C(s)}{D(s)} = \frac{s}{Js^3 + bs^2 + K_p s + \frac{K_p}{T_i}}$$

For this system to be stable, the real parts of the roots of the denominator must be positive. Again here, the error of the system in the absense of input, i.e. *R(s)* = 0 is found to be -*C(s)*. Applying the final value theorem, the steady state error can be found as:

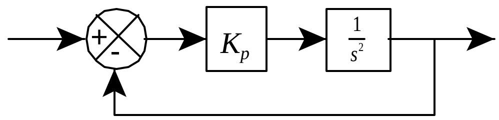
$$e_{ss} = \lim_{s \to 0}\left[s \frac{-s}{Js^3 + bs^2 + K_p s + \frac{K_p}{T_i}} \frac{A_d}{s} \right] = 0$$

BASIC CONTROL ACTIONS

Proportional-plus-Derivative Controllers

Consider the <u>proportionally controlled</u> second order inertial system shown below. The system transfer function is given by:

$$\frac{C(s)}{R(s)} = \frac{K_p}{Js^2 + K_p}$$



The roots of the denominator of the transfer function, i.e., the poles of the system are pure imaginary. This indicates that the system is at the limit of stability and would oscillate for ever.

If the proportional controller is replaced by a PD one, whose transfer function is given by:

$$K_p(1 + T_d s)$$

The transfer function for the system would be given by:

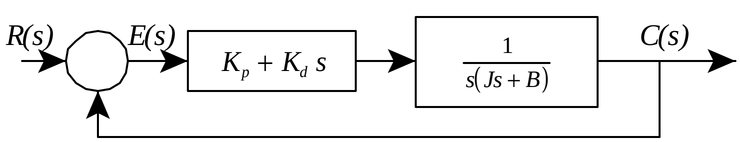
$$\frac{C(s)}{R(s)} = \frac{K_p(1 + T_d s)}{Js^2 + K_p T_d s + K_p}$$

BASIC CONTROL ACTIONS

Proportional-plus-Derivative Controllers

A study of the transfer functions of the inertial system when controlled by a P and PD controllers reveal that the introduction of the derivative part introduces damping into the system.

The effect of a PD controller on a second order system have been studied before, and its block diagram and transfer function are given below.



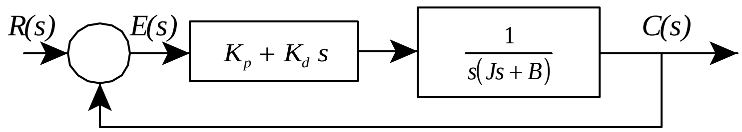
$$\frac{C(s)}{R(s)} = \frac{K_p + K_d s}{Js^2 + (B + K_d)s + K_p}$$

Note that the damping ratio and the natural frequency for this system are given respectively by:

$$x = \frac{B + K_d}{2\sqrt{K_p J}} \quad \text{and} \quad w_n = \sqrt{K_p / J}$$

Steady State Error of Second Order Systems with Proportional Plus Derivative Controller

For the second order system equipped with a proportional plus derivative controller as shown in the block diagram below;



Its transfer function is given by:

$$\frac{C(s)}{R(s)} = \frac{K_p + K_d s}{Js^2 + (B + K_d)s + K_p}$$

The error of the system is given by;

$$E(s) = \frac{Js^2 + Bs}{Js^2 + (B + K_d)s + K_p} R(s)$$

Steady State Error of Second Order Systems with Proportional Plus Derivative Controller

Hence the steady state error due to a step input is given by:

$$e_{ss} = \lim_{s \to 0} \left[s \frac{Js^2 + Bs}{Js^2 + (B + K_d)s + K_p} \frac{A_d}{s} \right]$$
$$= 0$$

And due to a unit ramp is given by:

$$e_{ss} = \lim_{s \to 0} \left[s \frac{Js^2 + Bs}{Js^2 + (B + K_d)s + K_p} \frac{A_d}{s^2} \right]$$
$$= 0$$