FREQUENCY RESPONSE ANALYSIS
Closed Loop Frequency Response

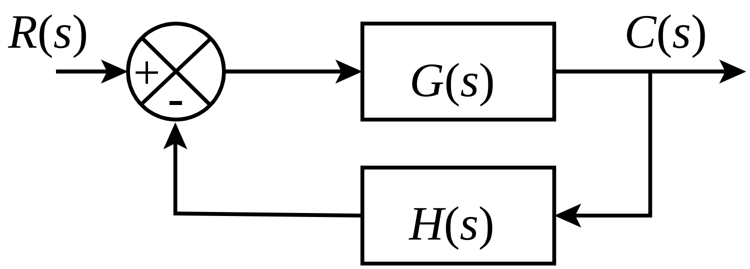
The Bode plot is generally constructed for an open loop transfer function of a system. In order to draw the Bode plot for a closed loop system, the transfer function has to be developed, and then decomposed into its poles and zeros. This process is tedious and cannot be carried out without the ais of a very powerful calculator or a computer.

Nichols, developed a simple process by which the unity feedback closed loop frequency response of a system can be easily deduced from the open loop transfer function. His approach will be outlined on the next few slides.

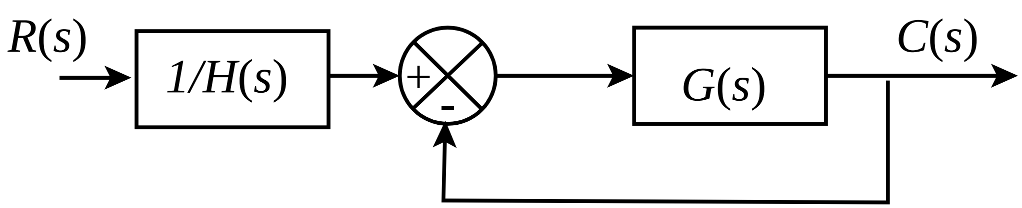
While the Nichols approach is derived strictly for a unity feedback system, any non-unity feedback system can be transformed to two cascaded open loop systems as follows:

FREQUENCY RESPONSE ANALYSIS
Closed Loop Frequency Response

Consider the following non unity feedback system:

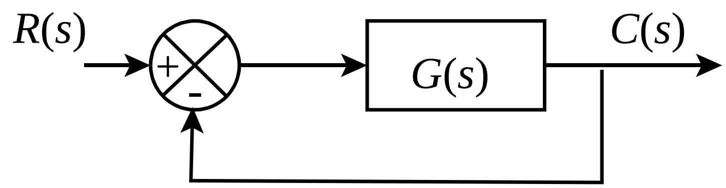


It can be transformed into the following system composed of a simple block cascaded to a unity feedback system:



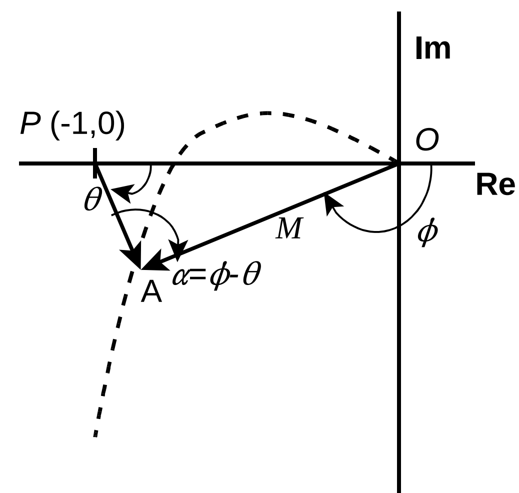
The frequency response can be then obtained using the additive feature of the Bode plots.

FREQUENCY RESPONSE ANALYSIS
Closed Loop Frequency Response



With reference to the generic unity feedback system block diagram and its polar plot; the system transfer function is given by:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

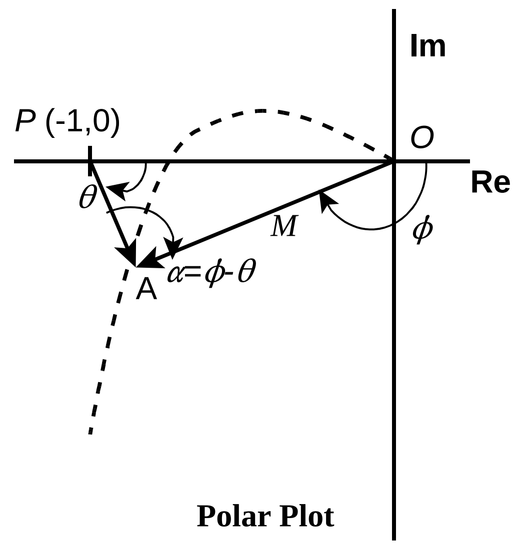


Polar Plot

The dashed line in the polar plot is the trace of the tip of the vector *OA* which represents the system. The length of the vector measure the magnitude of the system at a given frequency ω, and the angle ϕ represents the phase shift.

FREQUENCY RESPONSE ANALYSIS
Closed Loop Frequency Response

With reference to the polar plot;



$\overrightarrow{OA}$ represents $G(j\omega)$

$\overrightarrow{OP}$ represents -1

But $\overrightarrow{PA} = \overrightarrow{PO} + \overrightarrow{OA}$

$$= 1 + G(j\omega)$$

$$\therefore \frac{\overrightarrow{OA}}{\overrightarrow{PA}} = \frac{G(j\omega)}{1+G(j\omega)}$$

The phase shift angle of the closed loop system is the included angle between the vectors *OA* and *PA*, i.e.;

$$\alpha = \phi - \theta$$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Magnitude Locii

Since the open loop transfer function is a complex quantity that can be expressed as:

$$G(j\omega) = X + jY$$

It follows that the magnitude of the closed loop system can be expressed as follows:

$$M = \frac{|G(j\omega)|}{|1 + G(j\omega)|}$$

$$= \frac{|X + jY|}{|(1 + X) + jY|}$$

$$\therefore M^2 = \frac{X^2 + Y^2}{(1 + X)^2 + Y^2}$$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Magnitude Locii

Expanding and collecting terms, the previous equation gives:

$$X^2\left(1-M^2\right)-2M^2X-M^2+\left(1-M^2\right)Y^2=0$$

For M=1 the above equation reduces to;

$$X=-\tfrac{1}{2}$$

For $M\neq 1$, the equation can be written in the form:

$$X^2+\frac{2M^2}{M^2-1}X+\frac{M^2}{M^2-1}+Y^2=0$$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Magnitude Locii

The preceding equation canot be factored as it stands. However by adding;

$$\frac{M^2}{\left(M^2-1\right)^2}$$

to both sides and factoring gives:

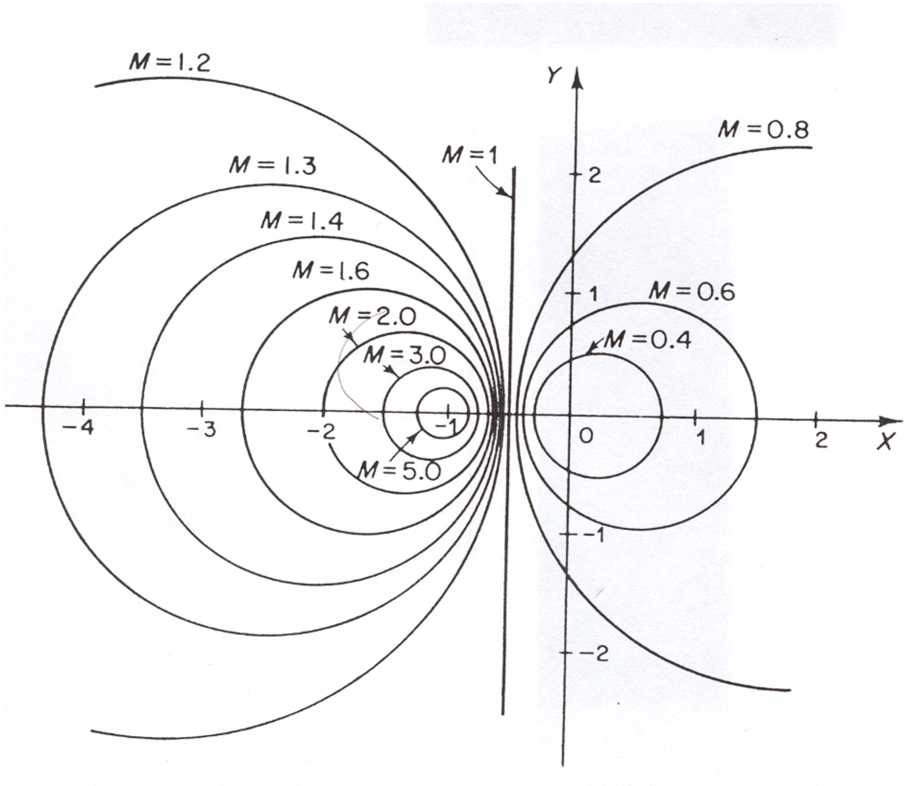
$$\left(X+\frac{M^2}{M^2-1}\right)^2+Y^2=\frac{M^2}{\left(M^2-1\right)^2}$$

This is an equation of a circle with radius and center as follows:

center: $\left(-\frac{M^2}{M^2-1},\,0\right)$ and radius: $\frac{M}{M^2-1}$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Magnitude Locii

The plot of the previous equation is shown below for different magnitudes *M*:



FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Phase Locii

The phase of the closed loop system can be expressed as follows:

$$\underline{\left| e^{j\alpha} \right.} = \underline{\left| \frac{X + jY}{1 + X + jY} \right.}$$

$$\therefore \alpha = \tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}$$

Now let $\alpha = N$, then;

$$N = \tan\left[\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X} \right]$$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Phase Locii

But

$$\tan\left(A-B\right)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$$\therefore N=\frac{\left(Y/X\right)-\left(Y/1+X\right)}{1+\left(Y/X\right)\left(Y/1+X\right)}$$

Simplifying gives:

$$N=\frac{Y}{X^2+X+Y^2}$$

or

$$X^2+X+Y^2-\frac{1}{N}Y=0$$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Phase Locii

The preceding equation canot be factored as it stands. However by adding;

$$\frac{1}{4}+\frac{1}{\left(2N\right)^2}$$

to both sides and factoring gives:

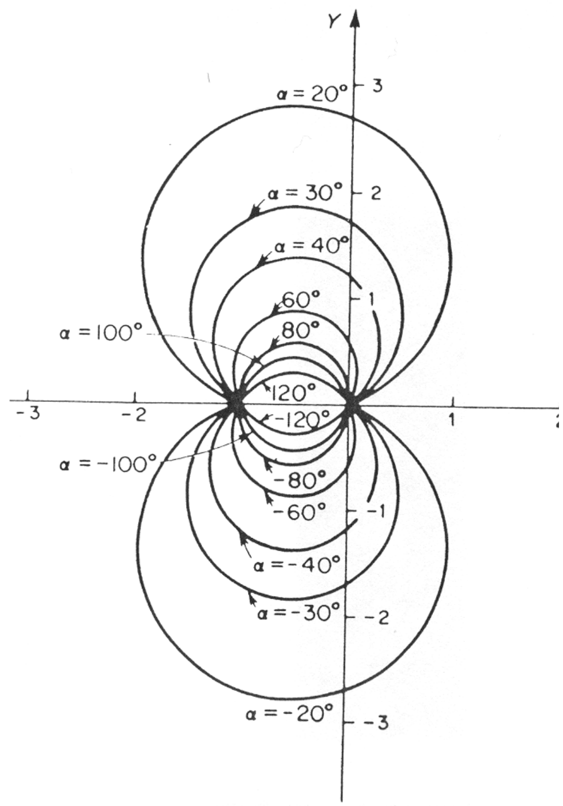
$$\left(X+\frac{1}{2}\right)^2+\left(Y-\frac{1}{2N}\right)^2=\frac{1}{4}+\left(\frac{1}{2N}\right)^2$$

This is an equation of a circle with radius and center as follows:

center: $\left(\frac{1}{2},\frac{1}{2N}\right)$ and radius: $\sqrt{\frac{1}{4}+\frac{1}{\left(2N\right)^2}}$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Phase Locii

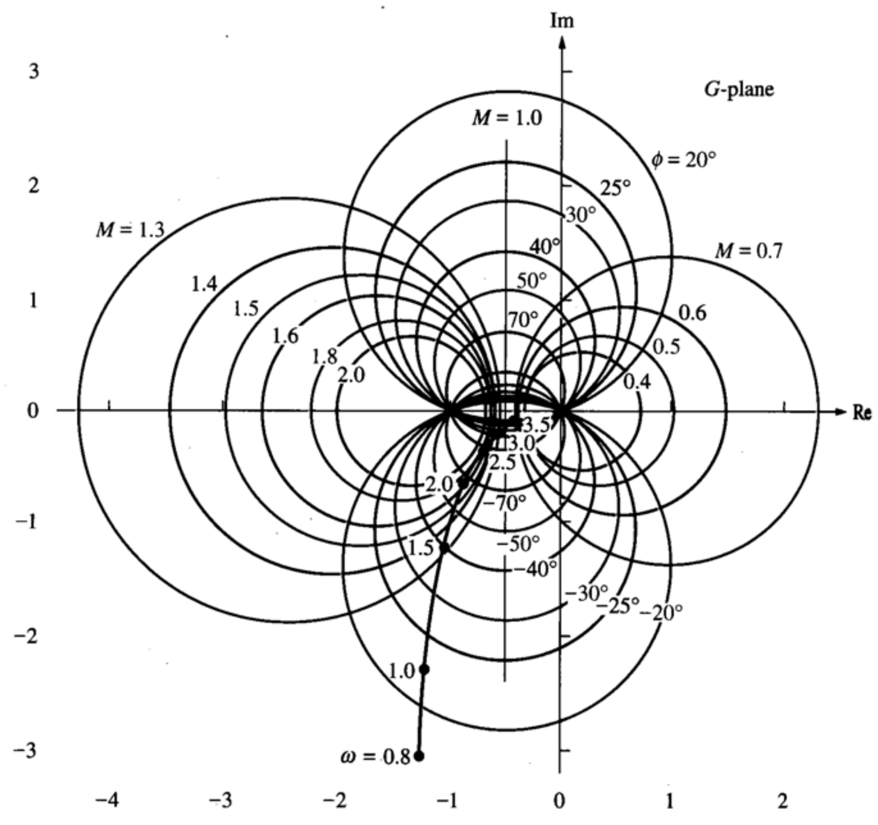
The plot of the previous equation is shown below for different phase angles α:



FREQUENCY RESPONSE ANALYSIS
Closed Loop Frequency Response from Open Loop Response

Using the constant magnitude and phase circles and the polar plot of the open loop to obtain the closed loop frequency response.

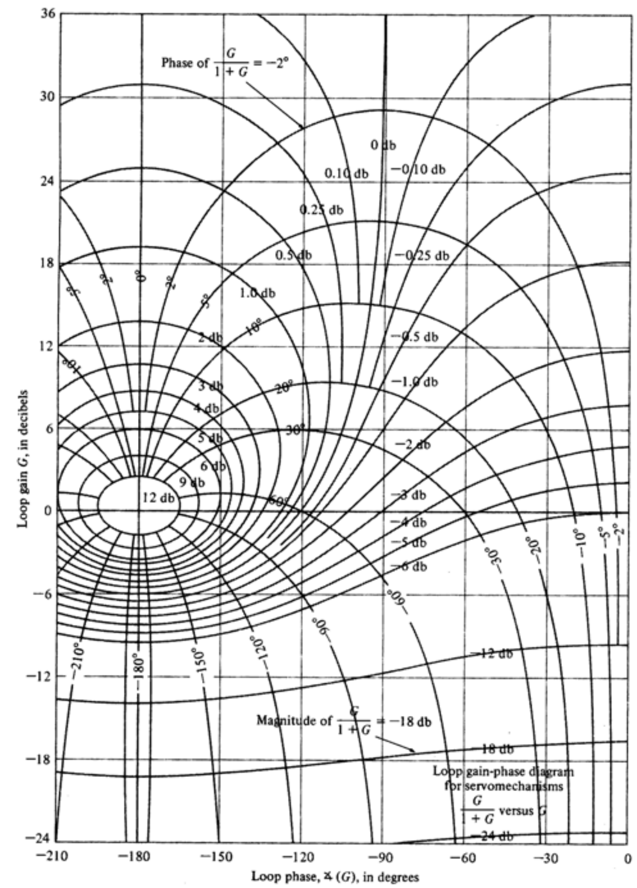
$$G(s) = \frac{50}{s(s+3)(s+6)}$$



FREQUENCY RESPONSE ANALYSIS
The Nichols Chart

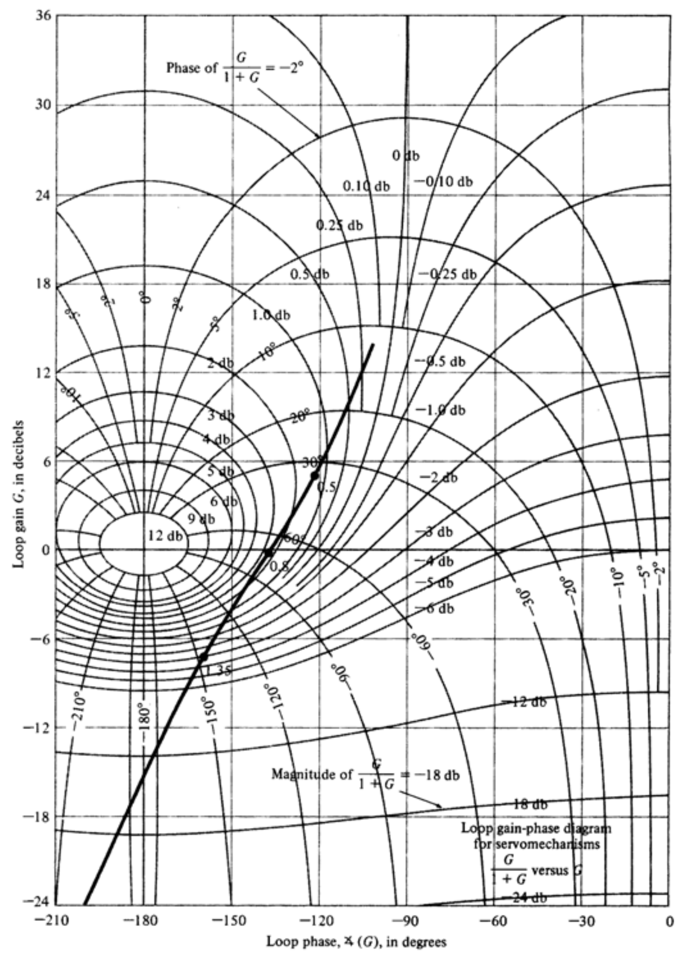
Nichols superimposed the log of the constant magnitude and phase circles on the log-magnitude phase plot to create the Nichols chart.

The magnitude and phase of the open loop values are plotted on the rectilinear log-magnitude phase grid, and the closed loop frequency response is read from the curvilinear grid.



FREQUENCY RESPONSE ANALYSIS
The Nichols Chart

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)(0.2j\omega+1)}$$



FREQUENCY RESPONSE ANALYSIS
The Nichols Chart

$$G(j\omega) = \frac{0.64}{j\omega\left(1 + j\omega - \omega^2\right)}$$